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Government expenditure multiplier under the zero lower bound: The role of public investment☆

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ABSTRACT

Under the zero lower bound the stimulating policies are conducted mostly by the fiscal authorities due to inefficiency of the traditional monetary policy. Recent research showed that in the borrower-saver framework the government expenditure multiplier can be significantly higher under the zero lower bound than under a positive interest rate. This paper explores the fiscal multiplier in the excess-savings liquidity trap in the extended framework which incorporates productive along with utility-enhancing government expenditures. The share of public investment in total expenditures and its productivity decrease the multiplier under the zero lower bound and increase it under a positive interest rate. In the former case, the higher share of liquidity-constrained borrowers weakens the negative effect of public investment, with an opposite impact in the latter case. An assumption about the two types of public expenditures intensifies the non-linear impact of the borrowers' share on the multiplier: the multiplier can become negative under zero lower bound for the sufficiently high share of borrowers.

1. Introduction

Under the zero lower bound, short-term interest rates have virtually stopped being the instrument of the monetary policy leaving the stabilizing economic measures to fiscal authorities. Although a wide range of theoretical research has investigated the size of fiscal multipliers, there is still a continuing debate on the size of the government expenditure multiplier under the zero lower bound. The two closest to the current research papers, Eggertsson and Krugman (2012), and Roulleau-Pasdeloup (2013), provide different evidence of the size of the multiplier in and out of the zero lower bound.

Eggertsson (2011) illustrates that the effect of macroeconomic policies is different under the zero lower bound (ZLB) and under a positive interest rate. In the former case the higher increase in the output can be obtained by policies aimed at stimulating aggregate demand (AD), not aggregate supply (AS). A cut in labor or capital income taxes, which are expansionary under a positive interest rate, can worsen the recession due to the upward-sloping demand curve under the ZLB. The results of Eggertsson (2011) were confirmed by the research in the DSGE framework (see e.g. Christiano, Eichenbaum, & Rebelo, 2011; Cogan, Cwik, Taylor, & Wieland, 2009; Erceg & Lindé, 2010) where a higher multiplier under the ZLB was also obtained. Eggertsson and Krugman (2012) focus on the government expenditure multiplier under the ZLB in the framework of heterogeneous agents (borrower-saver model). They obtain the multiplier of the utility-enhancing government

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expenditures which exceeds 1 under the ZLB. In this framework, apart from the standard demand-increasing effect of the public spending, government expenditures increase the output through the channel of liquidity-constrained borrowers who consume all their disposable income.

Meanwhile, Roulleau-Pasdeloup (2013) analyzes the government expenditure multiplier taking into account the structure of public spending: namely, productive and utility-enhancing types of government expenditures. The former was shown to gain a higher share in the total government expenditures during the recessions (Bachmann & Sims, 2012). As opposed to the results of Eggertsson and Krugman (2012) and Roulleau-Pasdeloup (2013) has found the fiscal multiplier to be lower in the case of an excess-savings liquidity trap than under a positive interest rate, showing that for a high share of productive spending in total expenditures private consumption is crowded out by productive government spending and the multiplier can even become negative.

I analyze the short-run government spending multiplier, extending the borrower-saver model of Eggertsson and Krugman (2012) to incorporate public investment. The developed framework, therefore, combines the introduction of debt constrained consumers which is known to increase the multiplier (Galí, López-Salido, & Vallés, 2007; Eggertsson & Krugman, 2012) with an assumption about productive government expenditures, that lower the multiplier under the ZLB (Roulleau-Pasdeloup, 2013).

The heterogeneity of the population coupled with public investment intensifies a non-linear impact of the borrowers' share on the multiplier. The share of borrowers increases the multiplier when there are no productive expenditures. However, the negative effect of productive spending under the ZLB is proportional to the borrowers' share, implying the ambiguity of its impact on the multiplier. As opposed to the results of Roulleau-Pasdeloup (2013), the multiplier under the ZLB remains higher than under a positive interest rate (3.76 and 0.31, respectively) due to the introduction of constrained agents, as they provide an additional transmission channel to the fiscal policy. The multiplier can still become negative under zero lower bound with a sufficiently high share of productive spending and for the high share of borrowers in the economy. However, this level of productive spending is higher than the one found by Roulleau-Pasdeloup (2013) and is decreasing with the share of borrowers.

There is no consensus in the empirical research about the value of the fiscal multiplier either. Empirical evidence suggests that private consumption tends to rise with government expenditure. Blanchard and Perotti (2002) show on the basis of structural VAR model that government spending increases private consumption. Moreover, the estimated government expenditure multiplier is always positive in the short-term and lower than 1.

Ramey (2011) extends Blanchard and Perotti (2002) by comparing the VAR and, Ramey and Shapiro (1998) narrative approach, which incorporates an impact of the anticipation of the fiscal policy, obtaining government expenditure multipliers between 0.6 and 1.2. However, Ramey (2011) found no evidence of the multiplier being higher under low interest rates. The other research extends Blanchard and Perotti (2002) by estimating the fiscal multiplier separately for the periods of expansions and recessions. As low interest rates are the consequence of the recession (Woodford, 2011), the results of this research are usually considered as an empirical evidence for the higher multiplier under the ZLB. Almunia, Bénétrix, Eichengreen, O Rourke, and Rua (2010) obtain the fiscal multipliers higher than 1 in the recession on the basis of panel data of 27 countries. Auerbach and Gorodnichenko (2012) on the basis of regime-switching SVAR model, obtain the positive multiplier that exceeds 1 in recessions and a smaller multiplier in expansions. While Owyang, Ramey, and Zubairy (2013), applying an approach of Ramey (2011) to the sample of two countries, the US and Canada, obtain for Canada a smaller multiplier under positive interest rates and long-run multipliers above 1 for the periods of recessions.

The results of this research can provide an explanation of the diverse empirical estimates of the government expenditure multiplier as both the structure of the government expenditures and the heterogeneity of the population affect the value of the fiscal multiplier. The results suggest that the government expenditure multiplier can still be higher in the ZLB then under a positive interest rate, even though the fiscal stimulus consists partly of the productive expenditures, if some consumers in the economy are liquidity-constrained.

Section 2 presents an extended model of Eggertsson and Krugman (2012), with the focus on two types of government expenditures. In Section 3 the government multiplier is derived under a positive interest rate and in the case of the zero lower bound. The effects of productive expenditures on the multiplier are compared in the two cases, specifying how these effects are changing with the share of borrowers. Section 4 summarizes the results of the research.

2. The model

The model of Eggertsson and Krugman (2012) with heterogeneous agents and standard utility-enhancing government expenditure was extended by incorporating productive government expenditures.

2.1. Households

Unit mass continuum of households consists of savers, χ_s, and borrowers, $1-\chi_s$. Agents differ in their time preference, $\beta(s)=\beta>\beta(b)$, where $\beta(i)\in(0,1)$ and $i=s,b$. Savers, which are standard Ricardian agents, smooth their consumption over time, while borrowers are liquidity-constrained and consume all their disposable income in each period, which allows us to consider them as standard Keynesian type agents.

Both types of consumers maximize an expected present value of instantaneous felicity, which is additively separable between consumption C_t, hours worked h_t and utility-enhancing government expenditures G_t^U.

$$E_0 \sum_{t=0}^{\infty} \beta^t(i) \left[U^i((C_t(i)) - \upsilon_t^i(h_t(i)) + \vartheta_t^i(G_t^U) \right], \; i = s, b \tag{1}$$

Eqs. (2)–(3) specify the consumption of differentiated good, represented by Dixit-Stiglitz aggregator, and the corresponding price index, where j is a type of a particular good.

$$C_t = \left(\int_0^1 c_t(j)^{(\theta-1)/\theta} dj \right)^{\theta/\theta-1}; \tag{2}$$

$$P_t = \left(\int_0^1 p_t(j)^{1-\theta} dj \right)^{1/\theta-1} \tag{3}$$

The demand function aggregated by two types of consumers is:

$$c_{jt} = C_t \left(\frac{p_{jt}}{P_t} \right)^{-\theta} \tag{4}$$

The aggregate per capita consumption in the economy is a weighted sum of per capita consumption of two types of agents:

$$C_t = \chi_s C_t^s + (1 - \chi_s) C_t^b \tag{5}$$

Intertemporal choice of the consumption and labor supply are defined by solving the utility maximization problem subject to the following budget constraint and debt limit constraint.

$$B_t^i(i) + W_t P_t h_t(i) + \int_0^1 \mathbb{P}(i) = (1 + i_{t-1}) B_{t-1}(i) + P_t C_t(i) + P_t T_t(i), \tag{6}$$

$$(1 + r_t) \frac{B_t(i)}{P_t} \le D_t. \tag{7}$$

Borrowing is realized by selling and purchasing of one period riskless nominal bonds, $B_t(i)$, with a nominal rate of return, i_t. Each agent also receives hourly real wage W_t paid for hours worked $h_t(i)$. Total income consists of the labor income and profit from the firm ownership, which is distributed equally among agents. The income is spent on consumption, on the repayment of the debt with interest (in the case of a borrower) and on the payment of lump-sum taxes, that can differ for each type of consumer.

Inequality (7) is the debt limit constraint. It postulates that the real value of debt including the real interest r_t can't exceed an exogenous real debt limit D_t.

It is assumed that for the savers, the debt limit constraint is not binding, so savers maximize their objective function subject to the above constraints with respect to consumption, hours worked and the amount lent. After solving this problem the following optimality conditions can be obtained:

$$U_c^s\left(C_t^s \right) = \beta \left(1 + i_t \right) E_t [U_c^s \left(C_{t+1}^s \right) \frac{P_t}{P_{t+1}}] \tag{8}$$

$$W_t = \frac{\upsilon_h^i(h_t^i)}{U_c^i(C_t^i)}, \; i = s, b \tag{9}$$

Eq. (8) sets the savers' optimal consumption path, while Eq. (9) specifies labor supply for both types of consumers.

For the borrowers, debt limit constraint is binding and their consumption is defined by the budget constraint in real terms, taking into account that debtors borrow up to the debt limit.

$$C_t^b = -D_{t-1} + \frac{D_t}{1 + r_t} + W_t^b h_t^b + \int_0^1 \mathbb{P}_t^b \frac{1}{P_t} - T_t^b \tag{10}$$

Consumers of this type spend all their disposable income plus the borrowed amount after returning the debt of the previous period plus interest.

2.2. Firms

There is a unit mass of firms, with a fraction λ setting prices each period, and $1 - \lambda$ setting prices one period in advance. Firms choose their optimal price by maximizing the present value of firm profits subject to the demand function and the production function.

$$E_t \sum_{t=0}^{\infty} \phi_t \left[p_t(j) y_t(j) - W_t P_t h_t(j) \right], \text{ s. t. } y_t(j) = Y_t \left(\frac{p_{jt}}{P_t} \right)^{-\theta}, y_t(j) = \left(G_t^P \right)^{\zeta} \left(h_t(j) \right)^{\eta}, \tag{11}$$

where $W_t = W(s)_t^{\chi_s} W(b)_t^{\chi_b}$, $\phi_t = \chi^s \phi_{1t}^s - (1 - \chi^s) \phi_{2t}^s$ – a discount factor and ϕ_{1t}^s, ϕ_{2t}^s are Lagrangian multipliers from utility maximization problem.

Production inputs of each type of good j are labor $h_t(j)$ and productive government spending. Productive spending G_t^P, which is included directly in the productive function, can be interpreted as an investment in the infrastructure.

2.3. Monetary and fiscal policy

Central bank follows Taylor rule with non negativity restriction on the nominal interest rate:

$$1 + i_t = \max\{1, (1 + r_t^n)(1 + \Pi_t)^{\phi_\pi}\}, \phi_\pi > 1 \tag{12}$$

where r_t^n is the natural interest rate, Π_t is an inflation rate.

The government chooses expenditures $G_t = G_t^P + G_t^U$ and sets taxes for both types of consumers. While in Eggertsson and Krugman (2012) only utility-enhancing government spending is considered, this model incorporates two types of government expenditures: utility-enhancing spending G_t^U and government investment G_t^P. Public spending is financed by taxes T_t^s and T_t^b, paid by savers and borrowers, respectively, and by issuing government bonds B_t^g which are bought by savers. No-arbitrage condition implies that an interest on public bonds i_t equals the interest on private borrowings.

Government budget constraint in real terms is:

$$G_t + D_{t-1}^g (1 + r_{t-1}) = D_t^g + T_t, \tag{13}$$

where $D_t^g = \frac{B_t^g}{P_t}$, $T_t = \chi_s T_t^s + (1 - \chi_s) T_t^b$.

Total government expenditures G_t has the same structure as private consumption, represented by the Dixit-Stiglitz aggregator with the corresponding demand function for a good j:

$$G_t = \left(\int_0^1 g_{jt}^{\theta - 1/\theta} dj \right)^{\theta/\theta - 1}, g_{jt} = G_t \left(\frac{p_{jt}}{P_t} \right)^{-\theta}. \tag{14}$$

The sequence of government expenditures, taxes and debt chosen by the government each period should satisfy the intertemporal budget constraint. The corresponding transversality condition is:

$$\lim_{N \to \infty} \frac{D_{t+N}^g}{(1 + r)^N} = 0$$

2.4. Equilibrium

Definition 1. Given the set of policy variables $\{G_t, T_t, D_t^g\}$ that satisfy the government budget constraint and the debt limit D_t, the set $\{W_t, r_t, Y_t, C_t, h_t\}$ defines equilibrium with $\pi_t = 0$, if it satisfies the optimality conditions of the agents and the equilibrium conditions for the goods market, bond market and labor market. However, according to Walras' Law only two market equilibrium conditions are needed to ensure equilibrium on these markets.

$$Y_t = \chi_s C_t^s + (1 - \chi_s) C_t^b + G_t \tag{15}$$

$$B_t^g + (1 - \chi_s) B_t^b = -\chi_s B_t^s \tag{16}$$

where $C_t = (1 - \chi_b) C_t^b + \chi_b C_t^b$, $W_t = W(s)_t^{\chi_s} W(b)_t^{\chi_b}$.

The model was log-linearized around the steady state with zero inflation. The steady state values are denoted by bar and the deviations from the steady state by hat. The steady state level of the real debt limit equals the low level of debt $\overline{D} = D^{low}$: the economy comes to the steady state after the debt limit falls from the high value to the low one. The model is summarized in Table 1.

Considering model in log-deviations from the steady state, it is convenient to represent two types of expenditures as shares of the total government spending $\widehat{G}_t = g_t^P + g_t^U$. Let $\psi \in (0, 1)$ be a share of government investment in total government spending, then $g_t^P = \psi \widehat{G}_t$ and $g_t^U = (1 - \psi) \widehat{G}_t$.

The Phillips curve has changed compared with Eggertsson and Krugman (2012) due to the introduction of productive government expenditures. Inflation level is now affected not only by total expenditure, but also by the productive part of government spending. The parameters κ and φ differ from those introduced in Eggertsson and Krugman (2012) to

Table 1
Summary of the Log-linear model.

Description	Analytical representation
Budget constraint, b	$\widehat{C_t^b} = \widehat{Y_t^b} + \beta\gamma_D \widehat{D}_t - \gamma_D \widehat{D_{t-1}} + \gamma_D \pi_t - \beta\gamma_D \left(i_t - E_t\pi_{t+1} - \bar{r} \right)$ MPSNOSPC
Euler equation, s	$\widehat{C_t^s} = E_t \widehat{C_{t+1}^s} - \sigma\left(i_t - E_t\pi_{t+1} - \bar{r} \right)$
Aggregate consumption	$\widehat{C}_t = \chi_s \widehat{C_t^s} + (1-\chi_s)\widehat{C_t^b}$
Labor supply, s	$\widehat{W}_t = \omega^s \widehat{h_t^s}(i) + \sigma^{s-1}\widehat{C_t^s}$
Labor supply, b	$\widehat{W}_t = \omega^b \widehat{h_t^b}(i) + \sigma^{b-1}\widehat{C_t^b}$
Labor market clearing	$\widehat{h}_t = \chi_s \widehat{h_t^s} + (1-\chi_s)\widehat{h_t^b}$
Production function	$\widehat{Y}_t = \eta h_t + \zeta g^P$
Phillips curve	$\pi_t = \kappa(1+\varphi)\widehat{Y}_t - \kappa\zeta(1+\varphi\sigma)g_t^P - \varphi\kappa\widehat{G}_t + E_{t-1}\pi_t$
Resource constraint	$\widehat{Y}_t = \widehat{C}_t + \widehat{G}_t = \widehat{C}_t + g_t^U + g_t^P = \zeta g_t^P + \eta \widehat{h}_t$
Taylor rule	$i_t = \max(0, r_t^n + \phi_\pi \pi_t)$
Government budget constraint	$\widehat{G}_t + \widehat{D_{t-1}^g}(1+\bar{r}) = \widehat{D_t^g} + \overline{D^g}\left(i_t - E_t\pi_{t+1} - \bar{r} \right) + \widehat{T}_t$

Note: $\varphi = \frac{\eta\sigma^{-1}}{\omega+1-\eta}$, $\widehat{C_t^i} = \log\frac{C_t^i}{\bar{Y}}$, $\widehat{h_t^i} = \log\frac{h_t^i}{\bar{Y}}$, $\omega^i = \frac{\bar{v}_{hh}^i \bar{h}}{\bar{v}_h^i} = \omega > 0$, $\sigma^i = -\frac{\bar{U}_c^i}{U_{cc}^i \bar{Y}} = \sigma > 0$, $i = s, b$. $\bar{r} = \log\beta^{-1}$, $\pi_t = \log\frac{P_t}{P_{t-1}}$, $i_t = \log\left(1 + i_t\right)$, $\gamma_D = \frac{\bar{D}}{\bar{Y}}$, $\overline{D^g} = \log\frac{\overline{D^g}}{\bar{Y}}$, $\kappa = \frac{\lambda(\omega+1-\eta)}{\eta(1-\lambda)}$,

incorporate the labor elasticity of the output.[1] Productive expenditures appeared separately in the Phillips curve due to their effect on the aggregate supply. The second term appears in the Phillips curve because increasing productive government expenditure raises marginal product of labor, decreasing, therefore, the marginal costs and, thus, prices.

3. Fiscal multiplier and comparative statics

3.1. Deleveraging shock

An unexpected deleveraging shock occurs in the short-term, while in the long-term, the economy returns to the steady state with a low value of debt limit. In order to analyze the effect of the shock, the model is rewritten, applying that all variables in the current period t are assumed to be the short-term variables, while the next period variables $t+1$ are at their long-term values.

In the long-run the output gap equals zero, $\widehat{Y}_L = 0$, with flexible price equilibrium $\pi_L = 0$ and the interest rate equals the natural interest rate, $i_L = r_L^n = \bar{r}$.

In the short-term the budget constraint and the disposable income of a borrower are:

$$\widehat{C_S^b} = \widehat{Y_S^b} - \widehat{D} + \gamma_D \pi_S - \gamma_D \beta\left(i_t - \pi_L - \bar{r} \right) - \widehat{T_S^b} \tag{18}$$

$$\widehat{Y_S^b} = \mu \widehat{Y}_S + \sigma^{-1}(\omega^{-1}\chi_b^{-1}\chi_s - 1)\widehat{G}_S - \tag{19}$$

$$-(1+\omega)\zeta\eta^{-1}g_S^P + \omega^{-1}\chi_b^{-1}\chi_s\sigma^{-1}(\widehat{C_L^s} - \sigma(i_S - E_t\pi_L - \bar{r})),$$

where $\widehat{D} = \frac{\beta D^{high} - \bar{D}}{\bar{Y}}$ and $\mu = (1+\omega^{-1})(\omega\eta^{-1} + \sigma^{-1}) - \sigma^{-1}\omega^{-1}\chi_b^{-1} > 0$.[2]

Thus, $\widehat{C_S^b}$ depends positively on the deviation of the disposable income and current inflation, which reduces the real value of debt. While the real interest rate paid for borrowings and the drop of the debt limit have a negative impact on consumption. Productive government expenditure also negatively affects the borrowers' consumption due to the substitution effect between public investment and labor. This negative effect decreases with η.

The consumption of the saver, given by Euler equation, in terms of short-term and long-term variables is:

$$\widehat{C}_S^s = \widehat{C}_L^s - \sigma(i_S - \pi_L - \bar{r}) \tag{20}$$

As opposed to the borrowers' consumption, savers' consumption depends on the expected future consumption and does not depend on the disposable income.

[1] Eggertsson and Krugman (2012) considered the one input production function linear in labor.
[2] For μ to be positive the following inequality should hold $\chi_b > \frac{\sigma^{-1}\omega^{-1}}{(1+\omega^{-1})(\omega\eta^{-1}+\sigma^{-1})}$, for the calibrated parameters $\chi_b > 0.24$

The Eq. (21) for aggregate demand is obtained by combining the resource constraint and the Eqs. (18) and (20), which specify the consumption of two types of agents in the short-term, taking into account that $\widehat{C}_L^s = 0$ and $\pi_L = 0$:

$$\hat{Y}_S = -\frac{\chi_s(\sigma+\omega^{-1})+\chi_b\gamma_D\beta}{1-\mu\chi_b}(i_S-\bar{r}) - \frac{\chi_b}{1-\mu\chi_b}\widehat{D} - \frac{\chi_b}{1-\mu\chi_b}\widehat{T}_S^b + + \frac{\chi_b\gamma_D}{1-\mu\chi_b}\pi_S + \frac{1+\sigma^{-1}\omega^{-1}\chi_s-\chi_b\sigma^{-1}}{1-\mu\chi_b}\widehat{G}_S - \frac{\left(1+\omega\right)\zeta\eta^{-1}\chi_b}{1-\mu\chi_b}g_S^P \tag{21}$$

Government spending affects the demand through the last two elements on the right-hand side of (21). While in Eggertsson and Krugman (2012) the government expenditure increases the output gap, in this model there is an additional negative impact of productive government spending on the aggregate demand. The negative impact of government investment disappears, when there are no borrowers in the economy, as it comes from an impact of this part of spending on borrowers' labor income. This effect decreases with the higher labor elasticity of the output, η, and increases with ζ.

Applying the definition of the natural interest rate to the Eq. (21), the IS curve can be written as:

$$\hat{Y}_S = -\frac{\chi_s(\sigma+\omega^{-1})+\chi_b\gamma_D\beta}{1-\mu\chi_b}(i_S-r_S^n) \tag{22}$$

In order to ensure that aggregate output falls with an increase in the nominal interest rate, an inequality $1-\mu\chi_b > 0$ should hold.[3] With a fall in the nominal interest rate, savers are encouraged to consume more. The higher consumption of savers results in the higher income of both savers and borrowers. Liquidity-constrained borrowers increase their demand as they consume all their additional income, which induces an increase in the output.

An interest rate in the flexible price equilibrium, the natural rate r_S^n, depends negatively on the value of the deleveraging shock. The fall of borrowers' consumption due to the lower debt limit lowers the natural interest rate, stimulating the consumption of savers.

$$r_S^n = \bar{r} - \frac{\chi_b}{\chi_s(\sigma+\omega^{-1})+\chi_b\gamma_D\beta}\widehat{D} + \frac{\gamma_D\chi_b}{\chi_s(\sigma+\omega^{-1})+\chi_b\gamma_D\beta}\pi_S - - \frac{\chi_b}{\chi_s(\sigma+\omega^{-1})+\chi_b\gamma_D\beta}\widehat{T}_S + \frac{1+\sigma^{-1}\omega^{-1}\chi_s-\chi_b\sigma^{-1}}{\chi_s(\sigma+\omega^{-1})+\chi_b\gamma_D\beta}\widehat{G}_S - \frac{(1+\omega)\zeta\eta^{-1}\chi_b}{\chi_s(\sigma+\omega^{-1})+\chi_b\gamma_D\beta}g_S^P \tag{23}$$

In the long-term, natural rate equals the steady state value $\bar{r}$. In the short-term it increases with inflation and government expenditure, while deleveraging shock and taxes diminish natural interest rate.

Taking into account the Taylor rule, two possible regimes emerge. In the case of a relatively small deleveraging shock, natural interest rate will remain positive and nominal interest rate adjusts to the new level of the debt limit, stimulating the demand and offsetting the negative impact of the shock. If the shock of the debt limit is high enough, natural interest rate can become negative and ZLB would become binding. In this case, output gap will be negative. Eggertsson and Krugman (2012) showed that in this case, the AD curve becomes upward-sloping, as inflation now increases output due to its negative effect on the real value of debt.[4] The AD curve remains upward-sloping in the model with productive expenditures, although it is steeper than in Eggertsson and Krugman (2012) due to the higher sensitivity of Y^b to the output ($\eta < 1$ in this case, as apposed to the case of one input production function).

The Eq. (24) for the equilibrium output gap was obtained from the combination of the Eq. (21) for the AD and the equation for the Phillips curve from Table 1:

$$\hat{Y}_S = -\frac{\chi_s(\sigma+\omega^{-1})+\chi_b\gamma_D\beta}{1-\chi_b(\mu+\gamma_D\kappa(1+\varphi)}(i_s-\bar{r}) - \frac{\chi_b}{1-\chi_b(\mu+\gamma_D\kappa(1+\varphi)}\widehat{D} - - \frac{\chi_b}{1-\chi_b(\mu+\gamma_D\kappa(1+\varphi))}\widehat{T}_S^b + + \frac{1+\sigma^{-1}\omega^{-1}\chi_s-\chi_b(\sigma^{-1}+\gamma_D\varphi\kappa)-\psi\zeta\chi_b(\kappa\gamma_D(1+\varphi\sigma)+(1+\omega)\eta^{-1})}{1-\chi_b(\mu+\gamma_D\kappa(1+\varphi))}\widehat{G}_S \tag{24}$$

Negative impact of productive government expenditure on the demand is supplemented by an additional negative impact of productive expenditure on prices. Both of them disappear, when there are no borrowers in the economy. Productive expenditure affects output gap, because the current consumption of borrowers depends positively on the current level of inflation, which decreases the real value of debt. Thus, this additional impact on the output disappears, when there are only savers in the economy.

Depending on the size of the deleveraging shock two cases can be considered: when nominal interest rate remains positive and when ZLB becomes binding and an the AD curve becomes upward-sloping.

[3] For this condition to be satisfied $\chi_b < 0.48$ based on the calibration presented below.

[4] The slope of the upward-sloping AD curve should be higher, than the slope of the AS curve, to ensure the equilibrium stability. Thus $(1-\mu\chi_b)/(\chi_b\gamma_D) > \kappa$ or $\chi_b < 1/(\kappa\gamma_D+\mu)$.

Table 2
Parameters.

Variable	Symbol	Value	Source
Discount factor	β	0.99	Standard
Response to inflation	ϕ_π	1.5	Standard
Elasticity of intertemporal substitution	σ	1	Eggertsson and Krugman (2012)
Inverse of Frisch labor supply elasticity	ω	1	Eggertsson and Krugman (2012)
Share of public investment	ψ	0.5	Roulleau-Pasdeloup (2013)
Production function elasticity	ζ	0.08	Roulleau-Pasdeloup (2013)
Debt share	γ_D	0.5	Eggertsson and Krugman (2012)
Share of firms with flexible prices	λ	0.5	Eggertsson and Krugman (2012)

The parameter values used for calibration are presented in Table 2. These values specify $\kappa = 1.1739$ and $\varphi = 0.4792$ as functions of the structural parameters. Calibration is based on the assumption of the constant return to scale, $\zeta + \eta = 1$.

3.2. Fiscal multiplier under a positive nominal interest rate

If the ZLB is not binding, the Central Bank follows the Taylor rule for the nominal interest rate:

$$i_t = r_t^n + \phi_\pi \pi_t, \quad \text{with } \phi_\pi > 1 \tag{25}$$

Under a positive interest rate, the government expenditure multiplier was obtained by substituting an Eq. (25) for a positive nominal interest rate and the Phillips curve in the IS curve Eq., (22):

$$\text{mult}_{PR}^{G} = \frac{\phi_\pi(\zeta\psi\kappa(1+\varphi\sigma)+\kappa\varphi)(\chi_s(\sigma+\omega^{-1})+\chi_b\gamma_D\beta)}{1-\mu\chi_b+\phi_\pi\kappa(1+\varphi)(\chi_s(\sigma+\omega^{-1})+\chi_b\gamma_D\beta)} \tag{26}$$

While in Roulleau-Pasdeloup (2013) a fiscal multiplier equals 0.57 when the ZLB is not binding and the economy consists only of Ricardian agents, in the case of heterogeneous agents multiplier equals 0.3148 for $\chi_b = 0.33$. Although the government multiplier is increasing with respect to the share of constrained agents, this estimate is lower than in the paper of Roulleau-Pasdeloup (2013).

The divergence of the current estimates and the results presented in Roulleau-Pasdeloup (2013) can be explained by the non-linear relationship of the multiplier and the share of borrowers. The value of the multiplier as a function of the share of borrowers under a positive interest rate is presented in Fig. 1a. The government expenditure multiplier is a hyperbolic

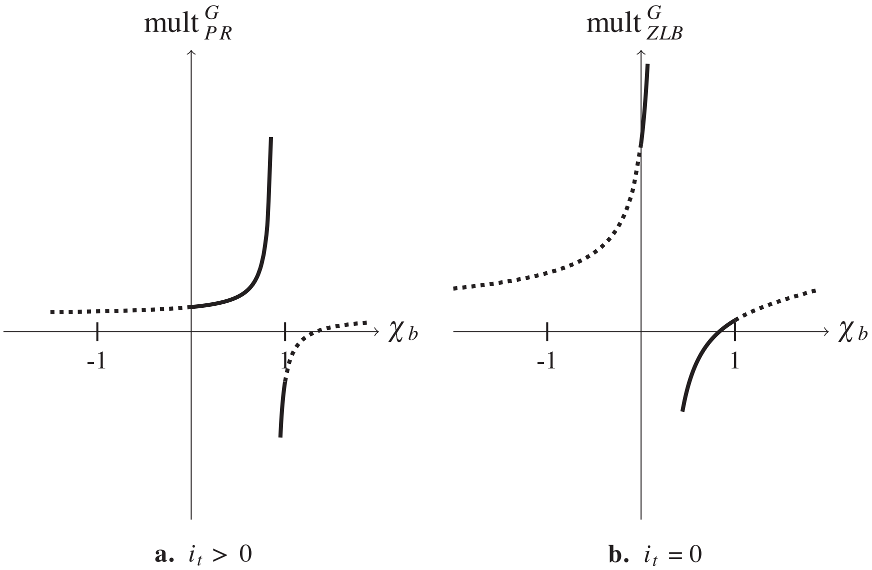

Fig. 1. Fiscal multiplier as a function of the share of borrowers.

function of χ_b with the asymptote at $\chi_b = 0.89$. Therefore, the multiplier is monotonically increasing with the share of borrowers for $\chi_b \in (0; 0.89)$.

Under a positive interest rate and, thus, a standard downward-sloping AD curve, the fiscal multiplier is increasing with price rigidity. An increase in the aggregate demand under a higher price rigidity would lead to lower inflation, which is contractionary under a positive interest rate. Moreover, an increase in the share of productive expenditure and its higher productivity lead to an increase in the multiplier, because higher public investment shifts the AS curve down, and this shift is expansionary in the normal case of a downward-sloping AD curve. Both these positive effects become stronger with an increase in the share of borrowers.

3.3. Fiscal multiplier under the ZLB

If the deleveraging shock is sufficiently high, the ZLB becomes binding due to the deflation effect. Output gap becomes:

$$\hat{Y}_S = \Gamma - \frac{\chi_b}{1-\chi_b(\mu+\gamma_D\kappa(1+\varphi))}\hat{D} - \frac{\chi_b}{1-\chi_b(\mu+\gamma_D\kappa(1+\varphi))}\widehat{T_S^b} + + \frac{1+\sigma^{-1}\omega^{-1}\chi_s - \chi_b(\sigma^{-1}+\gamma_D\varphi\kappa)}{1-\chi_b(\mu+\gamma_D\kappa(1+\varphi))}\widehat{G_S}$$
$$-\frac{\chi_b\zeta(\gamma_D\kappa(1+\varphi\sigma)+(1+\omega)\eta^{-1})}{1-\chi_b(\mu+\gamma_D\kappa(1+\varphi))}g_S^p,\ \Gamma = \frac{\chi_s(\sigma+\omega^{-1})+\chi_b\gamma_D\beta}{1-\chi_b(\mu+\gamma_D\kappa(1+\varphi))}\bar{r} \tag{27}$$

There are two effects of productive spending on the output: through total government expenditure and a separate negative effect.

Taking into account that $g_t^P = \psi\widehat{G_t}$ and assuming that an increase is financed by taxes on savers or debt, the government spending multiplier is:

$$\text{mult}_{ZLB}^G = \frac{1+\sigma^{-1}\omega^{-1}\chi_s - \chi_b(\sigma^{-1}+\gamma_D\varphi\kappa) - \psi\zeta\chi_b(\gamma_D\kappa(1+\varphi\sigma)+(1+\omega)\eta^{-1})}{1-\chi_b(\mu+\gamma_D\kappa(1+\varphi))} \tag{28}$$

Now we can compare the multiplier (28) with the one, obtained by Eggertsson and Krugman (2012). The numerator of (28) consists now of two items. The first one is the same as in Eggertsson and Krugman (2012), while the second item represents an additional negative effect that comes from a productive part of the government spending. This effect is a combination of the negative impact of public investment on the AD and AS. Both these effects decrease with a lower share of borrowers and disappear, when there are no borrowers in the economy. The negative effect on the price level comes from the negative effect of productive expenditure on the marginal cost. It affects the output gap through borrowers, as their current consumption depends positively on the current level of inflation, which is negatively affected by the production part of expenditure. The second negative component $(1+\omega)\zeta\eta^{-1}$ comes from an effect of productive expenditure on the wage and hours worked through the increased marginal product of labor, making workers more productive.

When the share of borrowers equals 0.33, the multiplier equals 3.76. Despite the introduction of productive government spending, the value of fiscal multiplier is higher under the ZLB than under a positive interest rate. This result is in line with the results of Eggertsson and Krugman (2012); Christiano et al. (2011), yet the higher value of the multiplier under ZLB contradicts the results of Roulleau-Pasdeloup (2013). This can be explained by the introduction of an additional channel, by which fiscal policy affects the output, through the presence of borrowers. This partially offsets the negative impact of the productive expenditure and ensures the higher impact of public expenditure on the AD than on the AS.

However, this additional increase in the output through the channel of borrowers is limited: under ZLB the multiplier is also a non-linear function of the share of borrowers (Fig. 1b). The borrowers' share below the threshold level increases the positive value of the multiplier. Further increase in the share of borrowers (for $\chi_b > 0.396$) can lead to the negative value of the multiplier. On the one hand, the multiplier is increasing with respect to the share of borrowers, when there is no productive spending, on the other hand, in the model with productive expenditure, its negative effect is proportional to χ_b and disappears when there are no borrowers in the economy.

Under the ZLB the multiplier can become negative for sufficiently high share of borrowers (for $\chi_b > 0.396$ when $\psi = 0.5$). However, for the share of borrowers which equals 0.33 the multiplier remains positive even for the highest share of productive spending, $\psi = 1$. Government multiplier would be positive if $1+\sigma^{-1}\omega^{-1}\chi_s - \chi_b(\sigma^{-1}+\gamma_D\varphi\kappa) > \psi\zeta\chi_b(\gamma_D\kappa(1+\varphi\sigma)+(1+\omega)\eta^{-1})$, as the denominator of the multiplier is assumed to be positive. This condition corresponds to the case when positive effect of productive spending on the output through the demand is higher than its effect on the AS.

In Roulleau-Pasdeloup (2013), consumption is crowded out by public spending if the productive part of government expenditure is higher than 0.64. In the model with the liquidity-constrained agents for this share of productive expenditure (0.64) to be sufficient for bringing the multiplier below zero, the share of borrowers should be higher than 0.82 which doesn't satisfy several restricting conditions on χ_b introduced earlier. Therefore, if an assumption about two types of consumers is taken into account the multiplier of government expenditure remains positive for any values of ψ, if the share of borrowers is low enough.

Under the ZLB both the share and the productivity of public investment have a negative impact on the value of the multiplier. A higher share of productive expenditure and its productivity lead to a bigger increase in AS (compared to the model without productive investment) which partially offsets an increase in the output provided by downward shift of the upward-sloping AD curve. As the result, a higher share of productive expenditure and its productivity decrease the magnitude of the multiplier. Roulleau-Pasdeloup (2013) has shown that in the case of the ZLB the fiscal multiplier is lower than under a positive interest rate (0.55 and 0.57, respectively) and can become even negative for a sufficiently high share of productive investment. Although the negative effect of productive investment remains in the framework considered here, the magnitude of the multiplier under the ZLB is still higher than under a positive interest rate due to the introduction of liquidity-constrained agents: 3.76 and 0.31, respectively.

4. Conclusion

This work incorporates productive government expenditure in a simple borrower-saver model to analyze how the presence of debt constrained agents affects the value of the multiplier when two types of government expenditure are considered: utility-enhancing and productive expenditure, which provides an additional increase in the aggregate supply (contractionary in the framework of zero lower bound). The introduction of borrowers, whose consumption depends on the current income, results in a higher value of the multiplier under zero lower bound, compared to the case when only Ricardian agents are considered.

A non-linear impact of the borrowers' share on the fiscal multiplier was revealed under a positive interest rate and under the ZLB. The non-linear relationship leads to the fact that the multiplier can become negative for sufficiently high share of productive investment in total government expenditure and for sufficiently high share of borrowers. This condition on the share of productive expenditure postulates that the positive effect of productive spending on the output through the demand is lower than its effect on the aggregate supply. The negative short-term effect of productive spending is coming from the negative impact that public investment has on the aggregate demand and on prices. Both these effects are decreasing with a lower share of borrowers and disappear, when there are no borrowers in the economy.

In the case of a positive nominal interest rate, both public investment productivity and its share in total expenditure, affect positively the fiscal multiplier, working in the same direction with the assumption about liquidity-constrained agents. Moreover, an increase in the share of borrowers strengthens the effect of production expenditure on the value of the multiplier. In the case of the zero lower bound, the situation is reversed: the share of productive expenditure and its productivity tend to lower the fiscal multiplier in the short term, which stands in favor of old Keynesian wasteful government expenditures. Although the negative effect of productive expenditure can be partly compensated by the higher number of borrowers in the economy up to the threshold after which the multiplier becomes negative.

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